

Impact of Decision Rules and Non-Cooperative Behaviors on Minimum Consensus Cost in Group Decision Making

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Abstract: In group decision making (GDM), it is sensible to achieve minimum consensus cost (MCC) because the consensus reaching process (CRP) resources are often limited. In this endeavour, though, there are still two issues that require paying attention to: (1) the impact of decision rules, including decision weights and aggregation functions, on MCC; and (2) the impact of non-cooperative behaviors on MCC. Hence, this paper analytically reveals the decision rules to minimize MCC or maximize MCC. Furthermore, detailed simulation experiments show the joint impact of non-cooperative behavior and decisions rules on MCC, as well as revealing the effect of the consensus within the established MCC target.

Keywords: Group decision making; Consensus; Cost; Decision rules; Non-cooperative behaviors; Simulation experiment.

1. Introduction

Group decision-making (GDM) can be viewed as a process where a group of decision makers express their opinions and aim at achieving a collective solution. Consensus reaching process (CRP) is a key issue in GDM by which decision makers are assisted to achieve consensus regarding certain collective solution. Traditionally, “hard” consensus in GDM considers only a lack of consensus state or a state of full or unanimous, which is inconvenient and unnecessary in real life ([Kacprzyk et al., 1997, 2010](#); [Kacprzyk & Zadrozny, 2010, 2016](#)). Thus, different consensus reaching processes (CRPs) based on a “soft” consensus measure in GDM have been proposed, with some excellent reviews on CRP available in ([Herrera-Viedma et al., 2014](#); [Palomares et al., 2014a](#)). In particular, the minimum consensus cost (MCC) and non-cooperative behaviors are becoming hot topics in CRPs.

Generally, preference changing are associated a cost and the resources for consensus building are limited ([Ben-Arieh & Easton, 2007](#); [Dong et al., 2010, 2015](#); [Dong & Xu, 2016](#); [Gong et al., 2017, 2019](#); [Tan et al., 2018](#)). Based on these premises, some CRPs with MCC have been developed ([Ben-Arieh & Easton, 2007](#); [Ben-Arieh et al., 2008](#); [Cheng et al., 2018](#); [Gong et al., 2015a, 2015b](#); [Zhang et al., 2017, 2019b](#)). Notably, [Zhang et al. \(2019c\)](#) investigated the consensus efficiency of existing CRPs with MCC and provided detailed simulation experiments based on the following comparison criteria: the number of adjusted decision makers; the number of adjusted alternatives; the number of adjusted preference values; the distance between the original and the adjusted preference information; and the number of negotiation rounds required to reach consensus.

In GDM problems, decision makers may behave uncooperatively by expressing dishonest opinions or refusing to change their opinions to favor their own profit. Two mainstream

research approaches have been developed in the literature to effectively address non-cooperative behaviors and ensure the quality of the GDM results: (1) managing non-cooperative behaviors in the aggregation process or selection process in the GDM, which focuses on the influence of the non-cooperative behaviors on the aggregation outcome (Dong et al., 2017; Pelta & Yager, 2010; Yager, 2001, 2002); (2) managing non-cooperative behaviors in the consensus process of the GDM, which mainly analyzes whether a consensus solution can be achieved under the presence of non-cooperative behaviors in the CRP (Dong et al., 2016, 2018b; Palomares et al., 2014b; Quesada et al., 2014).

Although numerous studies have been presented to analyze the MCC and non-cooperative behaviors in CRPs, there still exist two issues that need to be dealt with: the impact of the decision rules and of non-cooperative behaviors on MCC.

- (i) Impact of decision rules on MCC. In existing CRP studies, decision rules include decision weights and aggregation functions. It is natural that setting decision rules will lead to different consensus outcomes and will have an effect on the consensus cost. For example, in the selection of outstanding research projects, choosing different aggregation functions and decision weights can lead to different consensus outcome and cost. However, the existing research about MCC is to develop some consensus models under the given decision rules and decision weights, and it is still not clear how decision rules influence the minimum cost in reaching consensus in GDM.
- (ii) Impact of non-cooperative behaviors on MCC. Although non-cooperative behaviors have been extensively studied, the existing research about non-cooperative behaviors focuses on the management of non-cooperative behaviors in CRPs, and it is still unclear how non-cooperative behaviors influence on the MCC in CRPs. Therefore, it is necessary to reveal the internal mechanism of the non-cooperative behaviors impact on MCC.

This paper focuses on these two issues and it presents the following research contributions on the impact of decision rules and non-cooperative behaviors on MCC in GDM:

- (i) From a theoretical point of view, it is proved that the decision rule that minimizes MCC can be modeled with the ordered weighted average (OWA) with decision weight $w = (0.5, 0, \dots, 0, 0.5)^T$, while the decision rule that maximize the MCC is modeled with the OWA with decision weight $w = (1, 0, \dots, 0)^T$ or $w = (0, \dots, 0, 1)^T$.
- (ii) From an analytical point of view, Simulation experiments I, II and III are designed to show that the non-cooperative behaviors strongly increase the MCC. The MCC increases with an increase on the number of decision makers. This positive relationship is more obvious when decision makers are less tolerant to inconsistent views or the non-cooperative behaviors in the group are high. We also show non-cooperative behaviors is a more determiner factor in influencing the MCC than decision rules.

Furthermore, the effect of the consensus within the established target on MCC is also studied in the simulation experiments.

The rest of this paper is organized as follows: Section 2 describes the general CRP framework and the minimum-cost consensus model. Section 3 is devoted to the theoretical study to reveal of decision rules impact on MCC. Section 4 provides an analytical study with Simulation experiment I, II and III to further reveals the impact of non-cooperative behaviors and decision rules on MCC. Lastly, Section 5 concludes the paper.

2. Background

This section briefly describes the general CRP framework and the minimum-cost consensus model, which will provide the basis of this study.

2.1 The general CRP framework

The general CRP framework is depicted in Fig. 1 (Herrera-Viedma et al., 2002; Zhang et al., 2011). Once the individual experts provide their opinions, an aggregation function is carried out to derive the collective opinion. Then, the consensus level among the group of decision makers is measured by looking at the difference between the individual opinions and the collective opinion. If the consensus level is lower than an apriority acceptable consensus threshold value, then a feedback process is carried out to provide support to the decision makers to adjust their individual opinions with the aim of increasing the group consensus. This procedure is repeated for a maximum number of consensus rounds or until the group consensus level reaches the threshold level, whichever comes first.

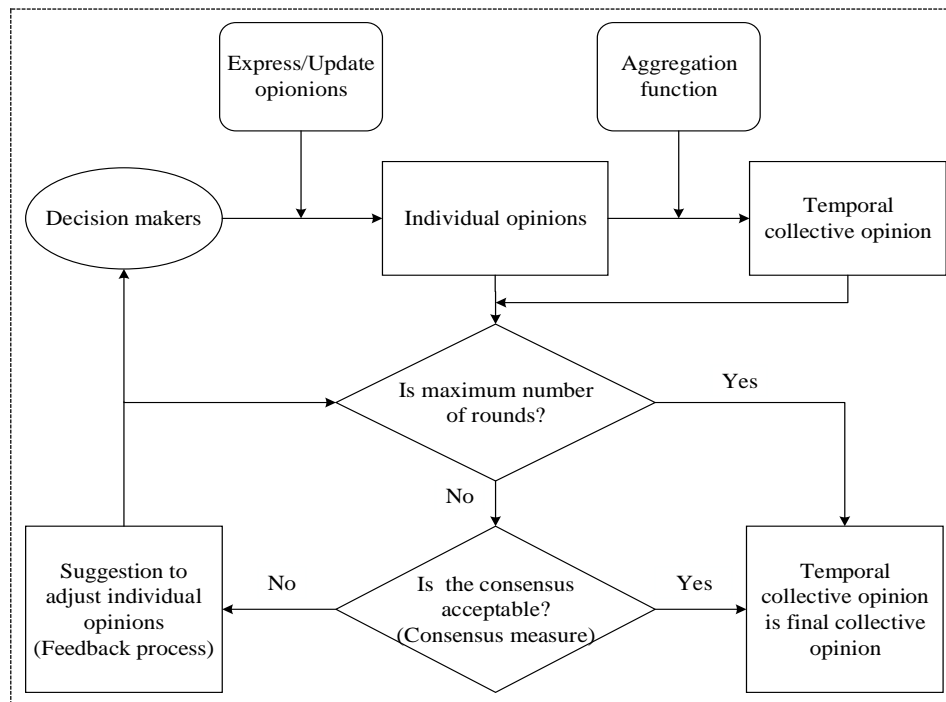


Fig. 1 Framework of the general CRP.

2.2 Minimum-Cost Consensus Model

In a GDM problem, let $D = \{d_1, d_2, \dots, d_n\}$ be the set of decision makers, $W = (w_1, w_2, \dots, w_n)^T$ the associated decision makers weights vector, where $w_i \geq 0$ ($i = 1, 2, \dots, n$) and $\sum_{i=1}^n w_i = 1$; $A = \{a_1, a_2, \dots, a_n\}$ be the set of individual opinions, where $a_i \in [0, 1]$ represents the decision maker d_i 's opinion on an alternative.

The minimum-cost consensus model includes three parts:

(1) *Consensus measurement*. Several methods can be used to measure the consensus level in a GDM problem (Fedrizzi, 1988; Kacprzyk et al., 1992). The most common is based on the use of a distance based measure, which is the one employed in the minimum-cost consensus model presented in (Chiclana et al., 2013). Let the collective opinion be represented by a^c . The consensus level of the decision maker d_i can be measured using the value $|a_i - a^c|$ ($i = 1, 2, \dots, n$). Let ε be a threshold value (Herrera-Viedma et al., 2002). If

$$|a_i - a^c| \leq \varepsilon, \text{ for all } i = 1, 2, \dots, n, \quad (1)$$

then all decision makers are considered to have reached an acceptable consensus. Otherwise, the decision makers need to adjust their opinions to increase the consensus level.

(2) *Consensus cost*. Let $\bar{a}_i \in \bar{A}$ denote the adjusted individual opinion of decision maker d_i , and let \bar{a}^c be the adjusted collective opinion. Ben-Arieh and Easton (2007) assumed that moving d_i 's opinion 1 unit has associated a cost $c_i \in [0, +\infty)$, and defined the linear consensus cost of moving d_i 's opinion from a_i to \bar{a}_i as $c_i |a_i - \bar{a}_i|$. Thus, it is natural to minimize the consensus cost, i.e.,

$$\min_{\bar{a}_i} \sum_{i=1}^n c_i |a_i - \bar{a}_i|. \quad (2)$$

(3) *Aggregation function*. In GDM problems, aggregation functions are used to fuse individuals' opinions to form a collective opinion (Akram et al., 2018, 2019a, 2019b, 2019c; Dong et al., 2010; Ogryczak & Śliwiński, 2003; Zhang et al., 2019a). The weighted average (WA) and ordered weighted average (OWA) operators are the most important aggregation functions in GDM problems (Dong et al., 2010; Ogryczak & Śliwiński, 2003; Zhang et al., 2011), which are defined by Eqs. (3) and (4), respectively.

$$F_{WA}(a_1, a_2, \dots, a_n) = \sum_{i=1}^n w_i a_i, \quad (3)$$

$$F_{OWA}(a_1, a_2, \dots, a_n) = \sum_{i=1}^n w_i a_{(i)}, \quad (4)$$

where $a_{(i)}$ is the i th largest element of $A = \{a_1, a_2, \dots, a_n\}$, and $W = (w_1, w_2, \dots, w_n)^T$ is the associated weight vector.

In minimum-cost consensus model, the WA and OWA operators are employed to aggregate individuals' opinion to obtain the collective decision option a^c , i.e.,

$$a^c = F_W(a_1, a_2, \dots, a_n). \quad (5)$$

Formally, Zhang et al. (2011) proposed the following minimum-cost consensus model with aggregation functions:

$$\begin{cases} \min_{\bar{a}_i} \sum_{i=1}^n c_i |a_i - \bar{a}_i| \\ \bar{a}^c = F_W(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) \\ |\bar{a}_i - \bar{a}^c| \leq \varepsilon, i = 1, 2, \dots, n \end{cases} \quad (6)$$

In this study, we denote this model as P_1 . Solving P_1 yields the optimal adjusted opinions. Similarly, the respective minimum-cost consensus models with aggregation functions OWA and WA, denoted models P_2 and P_3 , respectively, are:

$$\begin{cases} \min_{\bar{a}_i} \sum_{i=1}^n c_i |a_i - \bar{a}_i| \\ \bar{a}^c = F_{OWA}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) \\ |\bar{a}_i - \bar{a}^c| \leq \varepsilon, i = 1, 2, \dots, n \end{cases} \quad (7)$$

$$\begin{cases} \min_{\bar{a}_i} \sum_{i=1}^n c_i |a_i - \bar{a}_i| \\ \bar{a}^c = F_{WA}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n) \\ |\bar{a}_i - \bar{a}^c| \leq \varepsilon, i = 1, 2, \dots, n \end{cases} \quad (8)$$

3. Impact of decision rules on minimum consensus cost

In a CRP, the aggregation function and its associated decision weight play the role of the decision rules. This section reveals the impact of decision rules on MCC.

3.1 The decision rule to minimize MCC

The solution of the model P_4 below will be the optimal adjusted collective opinion that minimizes the MCC.

$$\begin{cases} \min_{\bar{a}^c} \min_{\bar{a}_i} \sum_{i=1}^n c_i |a_i - \bar{a}_i| \\ |\bar{a}_i - \bar{a}^c| \leq \varepsilon, i = 1, 2, \dots, n \end{cases} \quad (9)$$

Let model P_5 be defined by Eq. (10):

$$\begin{aligned} & \min_{\bar{a}_i} \sum_{i=1}^n c_i |a_i - \bar{a}_i| \\ & \bar{a}^c = (\min_i \bar{a}_i + \max_i \bar{a}_i) / 2 \\ & |\bar{a}_i - \bar{a}^c| \leq \varepsilon, i = 1, 2, \dots, n \end{aligned} \quad (10)$$

The following results show that the optimal solution of model P_5 is also the optimal solution of model P_4 .

Theorem 1: Let \bar{a}_i^* ($i = 1, 2, \dots, n$) and \bar{a}^{c*} denote the optimal adjusted individual opinions and adjusted collective opinion obtained by solving model P_5 . Then,

$\{\overline{a_1^*}, \overline{a_2^*}, \dots, \overline{a_n^*}, \overline{a^{c*}}\}$ is the optimal solution of model P_4 .

Proof:

Let $\{\overline{a_1}, \overline{a_2}, \dots, \overline{a_n}, \overline{a^c}\}$ be the optimal solution of model P_4 , and Ω_4 its feasible set. Similarly, let $\{\overline{a_1^*}, \overline{a_2^*}, \dots, \overline{a_n^*}, \overline{a^{c*}}\}$ be the optimal solution of model P_5 , and Ω_5 its feasible set.

Based on the condition $|\overline{a_i} - \overline{a^c}| \leq \varepsilon$ ($i = 1, 2, \dots, n$) of model P_4 , then we would get

$$\min_{\overline{a_i}} \sum_{i=1}^n c_i |a_i - \overline{a_i}| = \min_{\overline{a^c}} \left[\sum_{i: a_i < \overline{a^c} - \varepsilon} c_i (\overline{a^c} - \varepsilon - a_i) + \sum_{i: a_i > \overline{a^c} + \varepsilon} c_i (a_i - \varepsilon - \overline{a^c}) \right] \quad (11)$$

Also because

$$\begin{aligned} & \min_{\overline{a^c}} \min_{\overline{a_i}} \sum_{i=1}^n c_i |a_i - \overline{a_i}| \\ &= \min_{\overline{a^c}} \min_{\overline{a^c}} \left[\sum_{i: a_i < \overline{a^c} - \varepsilon} c_i (\overline{a^c} - \varepsilon - a_i) + \sum_{i: a_i > \overline{a^c} + \varepsilon} c_i (a_i - \varepsilon - \overline{a^c}) \right] \quad (12) \\ &= \min_{\overline{a^c}} \left[\sum_{i: a_i < \overline{a^c} - \varepsilon} c_i (\overline{a^c} - \varepsilon - a_i) + \sum_{i: a_i > \overline{a^c} + \varepsilon} c_i (a_i - \varepsilon - \overline{a^c}) \right] \end{aligned}$$

Then it will be

$$\min_{\overline{a^c}} \min_{\overline{a_i}} \sum_{i=1}^n c_i |a_i - \overline{a_i}| = \min_{\overline{a_i}} \sum_{i=1}^n c_i |a_i - \overline{a_i}|. \quad (13)$$

Therefore, model P_4 is equivalent to Eq. (14)

$$\begin{cases} \min_{\overline{a_i}} \sum_{i=1}^n c_i |a_i - \overline{a_i}| \\ |\overline{a_i} - \overline{a^c}| \leq \varepsilon, i = 1, 2, \dots, n \end{cases}. \quad (14)$$

Thus, $\{\overline{a_1}, \overline{a_2}, \dots, \overline{a_n}, \overline{a^c}\}$ and Ω_4 also represent the optimal solution and the feasible set of Eq. (14). Therefore, $\Omega_5 \subseteq \Omega_4$, and we have that:

$$\begin{aligned} \sum_{i=1}^n c_i |a_i - \overline{a_i}| &= \min_{\{\overline{a_1}, \overline{a_2}, \dots, \overline{a_n}, \overline{a^c}\} \in \Omega_4} \sum_{i=1}^n c_i |a_i - \overline{a_i}| \\ &\leq \min_{\{\overline{a_1}, \overline{a_2}, \dots, \overline{a_n}, \overline{a^c}\} \in \Omega_5} \sum_{i=1}^n c_i |a_i - \overline{a_i}| \quad (15) \\ &= \sum_{i=1}^n c_i |a_i - \overline{a_i^*}|. \end{aligned}$$

Moreover, the relationship between $\overline{a^c}$ and $\overline{a_i}$ is:

$$\overline{a_i} = \begin{cases} \overline{a^c} - \varepsilon, & i \in \{i: a_i \leq \overline{a^c} - \varepsilon\} \\ \overline{a^c} + \varepsilon, & i \in \{i: a_i \geq \overline{a^c} + \varepsilon\} \\ a_i, & i \in \{i: \overline{a^c} - \varepsilon < a_i < \overline{a^c} + \varepsilon\} \end{cases}, \quad (16)$$

which satisfies $\max_i \overline{a_i} - \min_i \overline{a_i} \leq 2\varepsilon$.

Furthermore, it is $|\overline{a_i} - \left[(\max_i \overline{a_i} + \min_i \overline{a_i}) / 2 \right]| \leq \varepsilon$ for all $i = 1, 2, \dots, n$. Therefore, $\{\overline{a_1}, \overline{a_2}, \dots, \overline{a_n}, \left[(\max_i \overline{a_i} + \min_i \overline{a_i}) / 2 \right]\} \in \Omega_5$. Consequently,

$$\begin{aligned}\sum_{i=1}^n c_i |a_i - \bar{a}_i| &\geq \min_{\{\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n, \bar{a}^c\} \in \Omega_5} \sum_{i=1}^n c_i |a_i - \bar{a}_i| \\ &= \sum_{i=1}^n c_i |a_i - \bar{a}_i^*|.\end{aligned}\quad (17)$$

From Eqs. (15) and (17), it is:

$$\sum_{i=1}^n c_i |a_i - \bar{a}_i| = \sum_{i=1}^n c_i |a_i - \bar{a}_i^*|. \quad (18)$$

Thus, $\{\bar{a}_1^*, \bar{a}_2^*, \dots, \bar{a}_n^*, \bar{a}^{c*}\}$ is the optimal solution to model P_4 .

This completes the proof of **Theorem 1**.

In the following, we further derive the decision rule between the optimal adjusted collective opinion and the optimal individual opinions of model P_4 . Based on **Theorem 1**, $\{\bar{a}_1^*, \bar{a}_2^*, \dots, \bar{a}_n^*, \bar{a}^{c*}\}$ is the optimal solution of model P_4 , which still satisfies the functional relationship $\bar{a}^{c*} = (\min_i \bar{a}_i^* + \max_i \bar{a}_i^*)/2$ of model P_5 , i.e. the decision rule that minimizes the MCC of model P_4 is:

$$\bar{a}^c = F_{OWA}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n), \quad (19)$$

with decision weight vector

$$OWA_min = (0.5, 0, \dots, 0, 0.5)^T. \quad (20)$$

3.2 The decision rule to maximize MCC

Solving the next model P_6 will lead to the optimal adjusted collective opinion that maximizes the MCC.

$$\begin{cases} \max_{\bar{a}^c} \min_{\bar{a}_i} \sum_{i=1}^n c_i |a_i - \bar{a}_i| \\ |\bar{a}_i - \bar{a}^c| \leq \varepsilon, i = 1, 2, \dots, n \end{cases} \quad (21)$$

First, we provide the following result regarding the optimal solution of model P_6

Lemma 1: Let $\{o_1, o_2, \dots, o_n, o^c\}$ denote the optimal solution of model P_1 . Then, it is:

$$o^c \in [\min_i a_i, \max_i a_i].$$

Proof:

Since $\{o_1, o_2, \dots, o_n, o^c\}$ is the optimal solution of model P_1 . Let $\{b_1, b_2, \dots, b_n, b^c\}$, $\{e_1, e_2, \dots, e_n, e^c\}$, $\{f_1, f_2, \dots, f_n, f^c\}$, and $\{g_1, g_2, \dots, g_n, g^c\}$ denote feasible solutions of model P_1 , where $b^c \in [0, \min_i a_i]$, $e^c \in (\max_i a_i, 1]$, $f^c = \min_i a_i$ and $g^c = \max_i a_i$ and ε being a small enough number.

(i) For $b^c \in [0, \min_i a_i]$, then

$$MCC_1 = \sum_{i=1}^n c_i |a_i - b_i| = \sum_{i=1}^n c_i (a_i - b_i). \quad (22)$$

(ii) For $e^c \in \left(\max_i a_i, 1\right]$, then

$$MCC_2 = \sum_{i=1}^n c_i |a_i - e_i| = \sum_{i=1}^n c_i (e_i - a_i). \quad (23)$$

(iii) For $f^c = \min_i a_i$, then

$$MCC_3 = \sum_{i=1}^n c_i |a_i - f_i| = \sum_{i=1}^n c_i (a_i - f_i). \quad (24)$$

(iv) For $g^c = \max_i a_i$, then

$$MCC_2 = \sum_{i=1}^n c_i |a_i - g_i| = \sum_{i=1}^n c_i (g_i - a_i). \quad (25)$$

Based on Eqs. (22) and (24), it is

$$MCC_1 - MCC_3 = \sum_{i=1}^n c_i (a_i - b_i) - \sum_{i=1}^n c_i (a_i - f_i) = \sum_{i=1}^n c_i (f_i - b_i). \quad (26)$$

The feasible solutions $\{b_1, b_2, \dots, b_n, b^c\}$ and $\{f_1, f_2, \dots, f_n, f^c\}$ satisfy the condition $|b_1 - b^c| \leq \varepsilon$ and $|f_1 - f^c| \leq \varepsilon$ ($i = 1, 2, \dots, n$) of model P_I , then $f_i \geq f^c - \varepsilon$ and $b_i \leq b^c + \varepsilon$ (for all $i = 1, 2, \dots, n$), hence

$$MCC_1 - MCC_3 = \sum_{i=1}^n c_i (f_i - b_i) \geq \sum_{i=1}^n c_i (f^c - b^c - 2\varepsilon). \quad (27)$$

When ε tends to 0, $\sum_{i=1}^n c_i (f^c - b^c - 2\varepsilon)$ tends to $\sum_{i=1}^n c_i (f^c - b^c) > 0$ and therefore it is

$$MCC_1 - MCC_3 > 0. \quad (28)$$

Thus, $MCC_1 > MCC_3$.

Based on Eqs. (23) and (25), we have

$$MCC_2 - MCC_4 = \sum_{i=1}^n c_i (e_i - a_i) - \sum_{i=1}^n c_i (g_i - a_i) = \sum_{i=1}^n c_i (e_i - g_i). \quad (29)$$

The feasible solutions $\{e_1, e_2, \dots, e_n, e^c\}$ and $\{g_1, g_2, \dots, g_n, g^c\}$ satisfy the condition $|e_1 - e^c| \leq \varepsilon$ and $|g_1 - g^c| \leq \varepsilon$ ($i = 1, 2, \dots, n$) of model P_I , then $e_i \geq e^c - \varepsilon$ and $g_i \leq g^c + \varepsilon$ (for all $i = 1, 2, \dots, n$), hence

$$MCC_2 - MCC_4 = \sum_{i=1}^n c_i (e_i - g_i) \geq \sum_{i=1}^n c_i (e_i - g_i - 2\varepsilon). \quad (30)$$

Taking limits when ε tends to 0, we would get that

$$MCC_2 - MCC_4 \geq \sum_{i=1}^n c_i (e_i - g_i) > 0. \quad (31)$$

Thus, $MCC_2 > MCC_4$.

Accordingly, it is clear that $[0, \min_i a_i) \cup (\max_i a_i, 1]$ is excluded from the range of the optimal collective opinion o^c of model P_I . Therefore, it is concluded that $o^c \in \left[\min_i a_i, \max_i a_i\right]$.

This completes the proof of **Lemma 1**.

Theorem 2: Let $\{p_1, p_2, \dots, p_n, p^c\}$ denote the optimal solution of model P_6 . Then, it is:

$$p^c = \begin{cases} \max_i p_i, & 2 \sum_{i=1}^n c_i a_i - \left(\min_i a_i + \max_i a_i \right) \sum_{i=1}^n c_i < 0 \\ \min_i p_i, & 2 \sum_{i=1}^n c_i a_i - \left(\min_i a_i + \max_i a_i \right) \sum_{i=1}^n c_i > 0 \end{cases}. \quad (32)$$

Proof:

The process of proving Theorem 2 is divided into two steps.

Step 1: Proof $p^c \in [\min_i a_i, \max_i a_i]$.

The aim of solving model P_I is to obtain the optimal solution under the fixed decision rule, while solving model P_6 aims at obtaining the optimal adjusted collective opinion that maximizes the MCC, which equals to finding the optimal decision rule that maximizes the MCC based on the Eq. (5).

The MCC obtained with model P_I with this optimal decision rule is the largest of all MCC obtained with model P_I with any decision rule. Obviously, **Lemma 1** still applies to model P_6 because the optimal solution $\{p_1, p_2, \dots, p_n, p^c\}$ of model P_6 is also the optimal solution of model P_I with this optimal decision rule. Hence, it is concluded that $p^c \in [\min_i a_i, \max_i a_i]$.

Step 2: Proof $p^c \rightarrow \min_i p_i$ or $\max_i p_i$.

Let $\{h_1, h_2, \dots, h_n, h^c\}$, $\{f_1, f_2, \dots, f_n, f^c\}$, and $\{g_1, g_2, \dots, g_n, g^c\}$ denote feasible solutions of model P_6 , where $h^c \in (\min_i a_i, \max_i a_i)$, $f^c = \min_i a_i$ and $g^c = \max_i a_i$ and ε is a small enough number.

(i) For $h^c \in (\min_i a_i, \max_i a_i)$, it is

$$MCC_1 = \sum_{i=1}^n c_i |a_i - h_i| = \sum_{i: h_i \geq a_i} c_i (h_i - a_i) + \sum_{i: h_i < a_i} c_i (a_i - h_i). \quad (33)$$

(ii) For $f^c = \min_i a_i$, it is

$$MCC_2 = \sum_{i=1}^n c_i |a_i - f_i| = \sum_{i=1}^n c_i (a_i - f_i) = \sum_{i: h_i \geq a_i} c_i (a_i - f_i) + \sum_{i: h_i < a_i} c_i (a_i - f_i). \quad (34)$$

(iii) For $g^c = \max_i a_i$, it is

$$MCC_3 = \sum_{i=1}^n c_i |a_i - g_i| = \sum_{i=1}^n c_i (g_i - a_i) = \sum_{i: h_i \geq a_i} c_i (g_i - a_i) + \sum_{i: h_i < a_i} c_i (g_i - a_i) \quad (35)$$

Based on Eqs. (33) and (34), it is

$$\begin{aligned} MCC_2 - MCC_1 &= \sum_{i: h_i \geq a_i} c_i (2a_i - f_i - h_i) + \sum_{i: h_i < a_i} c_i (h_i - f_i) \\ &\geq \sum_{i: h_i \geq a_i} c_i (2h_i - f_i - h_i) + \sum_{i: h_i < a_i} c_i (h_i - f_i) \\ &\geq \sum_{i: h_i \geq a_i} c_i (h_i - f_i) + \sum_{i: h_i < a_i} c_i (h_i - f_i) \\ &\geq \sum_{i=1}^n c_i (h_i - f_i). \end{aligned} \quad (36)$$

The feasible solutions $\{h_1, h_2, \dots, h_n, h^c\}$ and $\{f_1, f_2, \dots, f_n, f^c\}$ satisfy the conditions $|h_1 - h^c| \leq \varepsilon$ and $|f_1 - f^c| \leq \varepsilon$ ($i = 1, 2, \dots, n$) of model P_6 , then $h_i \geq h^c - \varepsilon$ and $f_i \leq$

$f^c + \varepsilon$ (for all $i = 1, 2, \dots, n$). Taking limits when ε tends to 0, we would get that

$$MCC_2 - MCC_1 \geq \sum_{i=1}^n c_i (h^c - f^c) > 0. \quad (37)$$

Based on Eqs. (33) and (35), it is

$$\begin{aligned} MCC_3 - MCC_1 &= \sum_{i:h_i \geq a_i} c_i (g_i - h_i) + \sum_{i:h_i < a_i} c_i (g_i + h_i - 2a_i) \\ &\geq \sum_{i:h_i \geq a_i} c_i (g_i - h_i) + \sum_{i:h_i < a_i} c_i (g_i + h_i - 2h_i) \\ &\geq \sum_{i:h_i \geq a_i} c_i (g_i - h_i) + \sum_{i:h_i < a_i} c_i (g_i - h_i) \\ &\geq \sum_{i=1}^n c_i (g_i - h_i). \end{aligned} \quad (38)$$

The feasible solutions $\{h_1, h_2, \dots, h_n, h^c\}$ and $\{g_1, g_2, \dots, g_n, g^c\}$ satisfy the conditions $|h_1 - h^c| \leq \varepsilon$ and $|g_1 - g^c| \leq \varepsilon$ ($i = 1, 2, \dots, n$) of model P_6 , then $h_i \leq h^c + \varepsilon$ and $g_i \geq g^c - \varepsilon$ (for all $i = 1, 2, \dots, n$). Similarly, it would be

$$MCC_3 - MCC_1 \geq \sum_{i=1}^n c_i (g^c - h^c) > 0. \quad (39)$$

From Eqs. (37) and (39), it can be concluded that $MCC_2 > MCC_1$ and $MCC_3 > MCC_1$ when ε is a small enough number, i.e. the closer p^c is to $\min_i a_i$ or $\max_i a_i$, the larger the MCC is. Because $p^c \in [\min_i p_i, \max_i p_i]$, this conclusion are guaranteed in the limit case, i.e. $p^c = \min_i p_i$ when $p^c \rightarrow \min_i a_i$ and $p^c = \max_i p_i$ when $p^c \rightarrow \max_i a_i$. Therefore, we have

$$p^c = \begin{cases} \max_i p_i, & MCC_3 > MCC_2 \\ \min_i p_i, & MCC_3 < MCC_2 \end{cases} \quad (40)$$

Taking limits when ε tends to 0, we would get that

$$p^c = \begin{cases} \max_i p_i, & 2 \sum_{i=1}^n c_i a_i - (\min_i a_i + \max_i a_i) \sum_{i=1}^n c_i < 0 \\ \min_i p_i, & 2 \sum_{i=1}^n c_i a_i - (\min_i a_i + \max_i a_i) \sum_{i=1}^n c_i > 0 \end{cases} \quad (41)$$

This completes the proof of **Theorem 2**.

Based on the **Theorem 2**, the decision rule that maximizes the MCC of model P_6 is

$$\overline{a^c} = F_{OWA}(\overline{a_1}, \overline{a_2}, \dots, \overline{a_n}), \quad (42)$$

with the following associated decision weight vector

$$OWA_{max} = \begin{cases} (1, 0, \dots, 0)^T, & 2 \sum_{i=1}^n c_i a_i - (\min_i a_i + \max_i a_i) \sum_{i=1}^n c_i < 0 \\ (0, \dots, 0, 1)^T, & 2 \sum_{i=1}^n c_i a_i - (\min_i a_i + \max_i a_i) \sum_{i=1}^n c_i > 0 \end{cases}. \quad (43)$$

4. Simulation analysis

In this section, we further reveal the impact of non-cooperative behaviors and decision rules on MCC via Simulation experiments I, II and III. In the minimum-cost consensus model, c_i denotes the cost of moving d_i 's opinion 1 unit, and in this study we argue that it can

represent the non-cooperative behavior coefficient of decision maker d_i because larger c_i values means less cooperation to reach consensus. Thus, $C = \{c_1, c_2, \dots, c_n\}$ is considered to be the set of non-cooperative behavior coefficients. Meanwhile, we argue that the decision rules consist of the aggregation function and the associated decision weight.

4.1 Simulation experiment I: Impact of non-cooperative behaviors on MCC

The basic idea of Simulation experiment I is to randomly generate original individual opinions and values of c_i ($i = 1, 2, \dots, n$). Particularly, c_i is randomly generated based on the normal distribution $N(\mu_c, \sigma_c^2)$ subject to the constraint $c_i > 0$, where μ_c and σ_c^2 approximately measure the expectation and variance of d_i 's non-cooperative behavior coefficient, respectively. Thus, we study the impact of non-cooperative behaviors on MCC through μ_c and σ_c^2 . Simulation experiment I can be formally presented as follows.

Simulation experiment I:

Input: The number of decision makers n , the expectation of d_i 's non-cooperative behavior coefficient μ_c , the variance of d_i 's non-cooperative behavior coefficient σ_c^2 , the established threshold value ε and decision weight $w = (w_1, w_2, \dots, w_n)^T$.

Output: The minimum consensus cost, MCC.

Step 1: Let $A = \{a_1, a_2, \dots, a_n\}$ be the original individual opinion set in model P_2 , where a_i ($i = 1, 2, \dots, n$) is randomly selected using the uniform distribution on $[0, 1]$.

Step 2: Let $C = \{c_1, c_2, \dots, c_n\}$ be the set of non-cooperative behavior coefficients in model P_2 , where c_i ($i = 1, 2, \dots, n$) is randomly generated based on $N(\mu_c, \sigma_c^2)$ subject to the constraint $c_i > 0$.

Step 3: Compute the MCC based on model P_2 .

Without loss of generality, in the simulation we consider two decision rules:

$$OWA_1 = (1/n, \dots, 1/n)^T \text{ and } OWA_2 = (0, 1/(n-2), \dots, 1/(n-2), 0)^T.$$

We set different values for n , ε , μ_c , σ_c^2 and run Simulation experiment I 10,000 times to calculate the average of the MCC values, which is shown in Figs. 2 and 3.

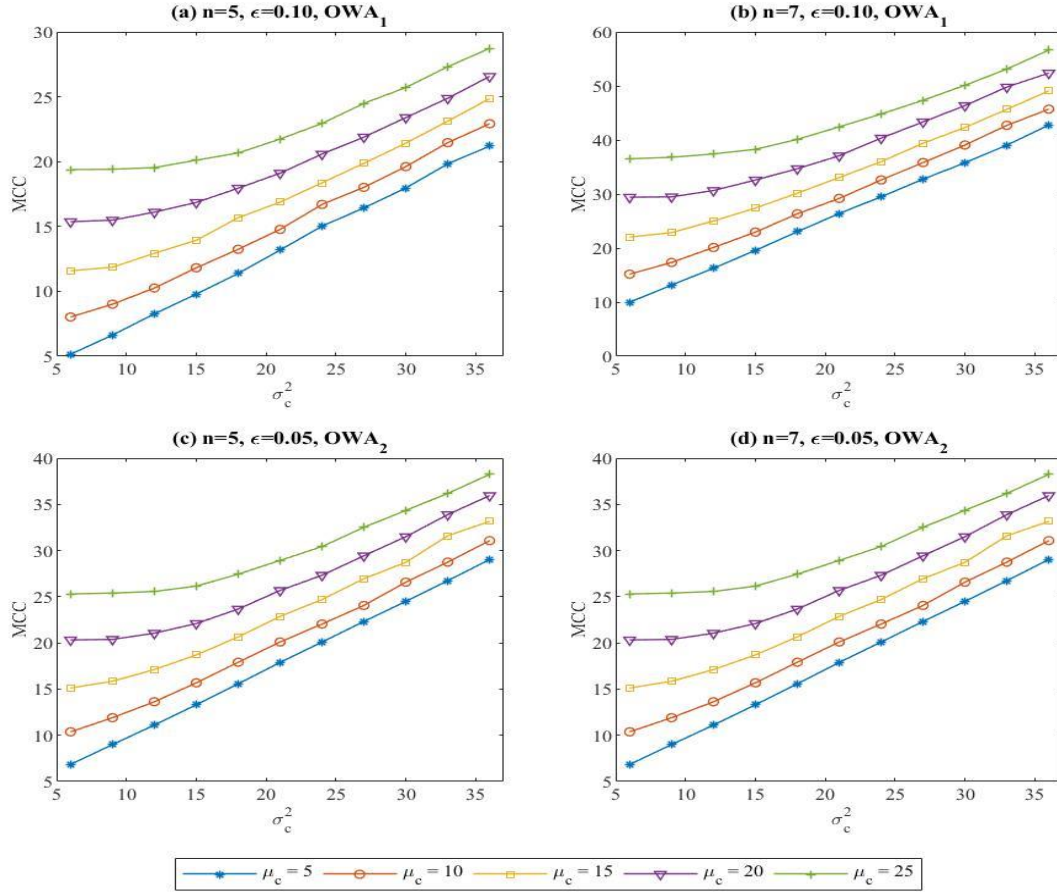


Fig. 2 Effects of μ_c and σ_c^2 on the MCC under different parameters

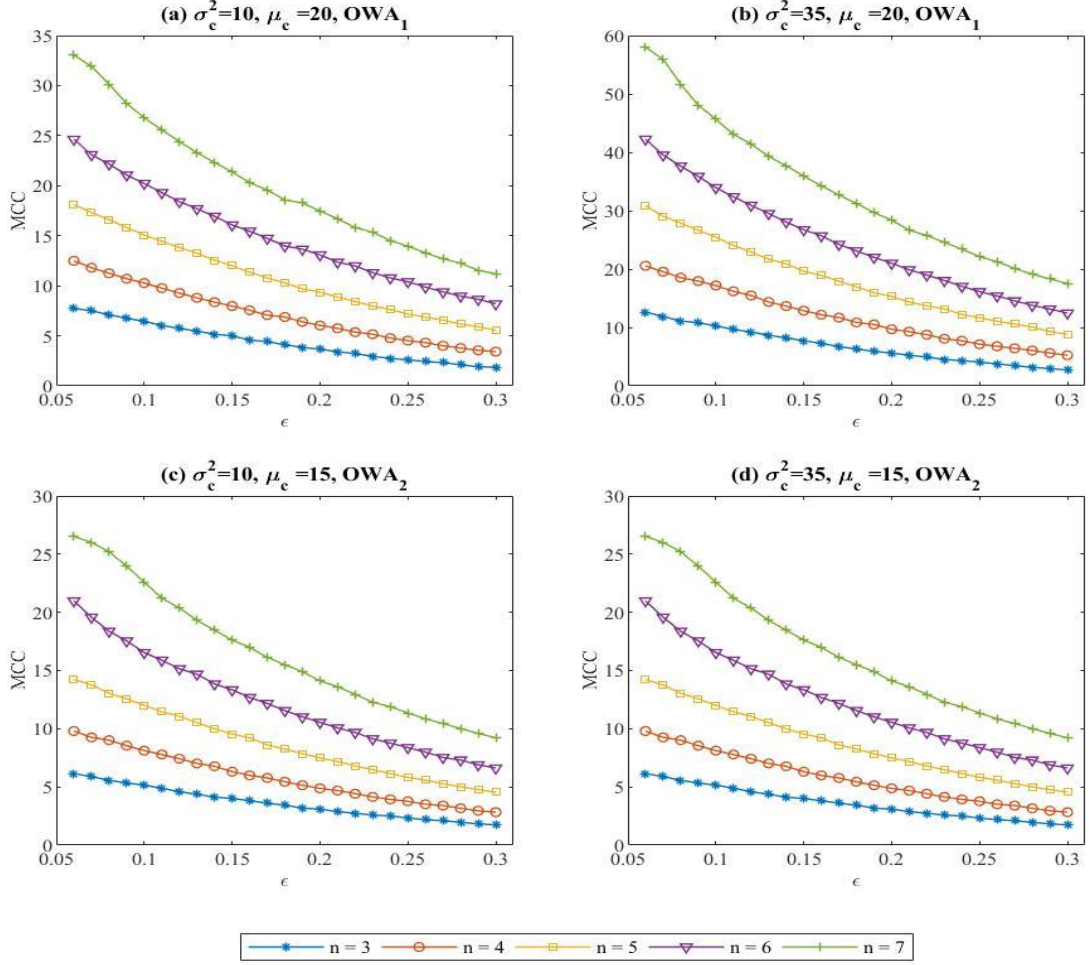


Fig. 3 Effects of ε and n on the MCC under different parameters

From Figs. 2 and 3, we draw the following observations:

(1) Fig. 2 shows that the MCC increases as μ_c and σ_c^2 increase under different parameters. This implies that the non-cooperative behaviors in the group, defined by μ_c and σ_c^2 , strongly increase the MCC, which translates in an increase in the difficulty when reaching consensus in GDM.

(2) Fig. 3 shows that the MCC increases as n increases under different parameters, being this increment more significant the smaller ε values or the larger σ_c^2 values are, respectively. This agrees with the real increase of difficulty in reaching consensus the larger the group experts. Meanwhile, the number of decision makers will have a stronger impact on the consensus reaching in GDM when decision makers are less tolerant to inconsistent views, defined by ε , or the larger the non-cooperative behaviors in the group are, than otherwise.

4.2 Simulation experiment II: The joint impact of non-cooperative behaviors and decisions rules on MCC

Let $C = \{c_1, c_2, \dots, c_n\}$ be the set of non-cooperative behavior coefficients, defined by unit cost; $S(C) = \sum_{i=1}^n c_i$ the sum of the non-cooperative behavior coefficients; and $D(C) =$

$(\sum_{i=1}^n [c_i - S(C)/n]^2)/n$ the variance of the non-cooperative behavior coefficients. Let $w_i = \bar{w}_i / \sum_{j=1}^n \bar{w}_j$ ($i = 1, 2, \dots, n$) and \bar{w}_i be randomly generated using normal distribution $N(0.2, \sigma_w^2)$ subject to the constraint $\bar{w}_i > 0$. Then, σ_w^2 approximately measures the variance of decision weights.

The basic idea of Simulation experiment II is to randomly generate original individual opinions, and the values of w_i ($i = 1, 2, \dots, n$). Then, we investigate the joint impact of non-cooperative behaviors and decisions rules on MCC through C and σ_w^2 . Simulation experiment II can be formally presented as follows.

Simulation experiment II:

Input: The number of decision makers n , the variance of decision weights σ_w^2 , the established threshold value ε and the non-cooperative behavior coefficient set $C = \{c_1, c_2, \dots, c_n\}$.

Output: The minimum consensus cost, MCC.

Step 1: Let $A = \{a_1, a_2, \dots, a_n\}$ be the original individual opinion set of model P_2 (or model P_3). where a_i ($i = 1, 2, \dots, n$) is randomly selected using the uniform distribution on $[0, 1]$.

Step 2: Let $W = (w_1, w_2, \dots, w_n)^T$ be decision weight of model P_2 (or model P_3), where $w_i = \bar{w}_i / \sum_{j=1}^n \bar{w}_j$ ($i = 1, 2, \dots, n$), and \bar{w}_i is randomly generated using normal distribution $N(0.2, \sigma_w^2)$ subject to the constraint $\bar{w}_i > 0$.

Step 3: Compute the MCC based on model P_2 (or model P_3).

In order to reflect the impact of $D(C)$ on MCC, we consider eight non-cooperative behavior coefficients sets:

$$C_1 = \{3, 3, 3, 3, 3\}, C_2 = \{5, 4, 3, 2, 1\}, C_3 = \{1, 2, 7, 3, 2\}, C_4 = \{8, 5, 2, 0, 0\},$$

$$C_5 = \{10, 2, 1, 1, 1\}, C_6 = \{1, 2, 3, 4, 5\}, C_7 = \{7, 3, 2, 1, 2\}, C_8 = \{0, 0, 8, 5, 2\},$$

with same sum $S(C_k) = 15$ ($k = 1, 2, \dots, 8$) but different variance values:

$$D(C_1) = 0, D(C_2) = 2, D(C_3) = 4.4, D(C_4) = 9.6,$$

$$D(C_5) = 12.4, D(C_6) = 2, D(C_7) = 4.4, D(C_8) = 9.6.$$

We set different σ_w^2 values and run Simulation experiment II 10,000 times to calculate the average of the MCC values, which are shown in Figs. 4 and 5.

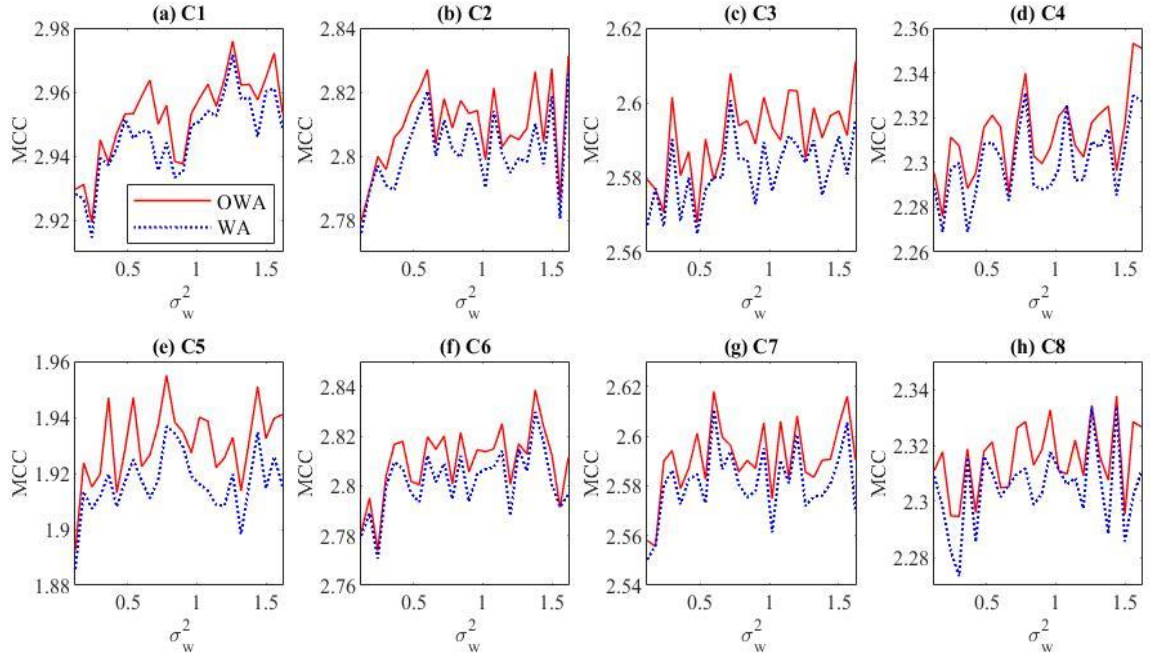


Fig. 4 The difference of models P_2 (OWA) and P_3 (WA) on MCC with $n = 5$ and $\epsilon = 0.05$

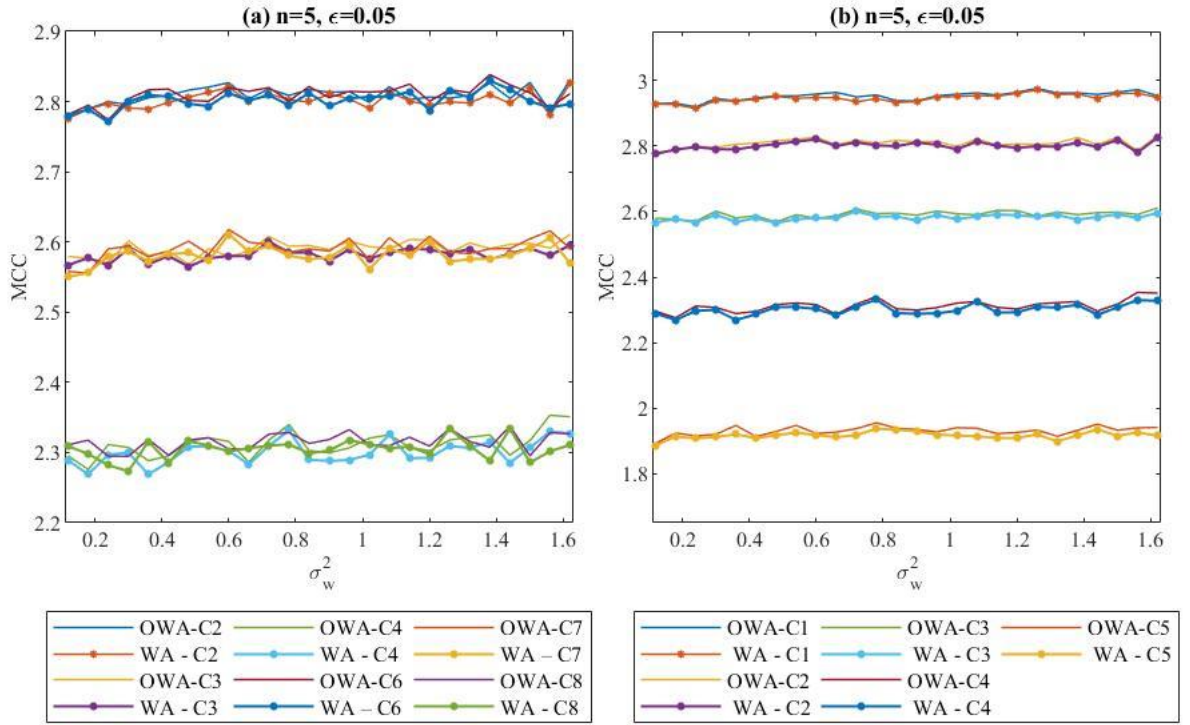


Fig. 5 The joint impact of σ_w^2 and $D(C)$ on MCC

From Figs. 4 and 5, we draw the following observations:

(1) Fig. 4 shows that the MCC obtained with model P_2 is larger than the one obtained with model P_3 under different parameters, which means that it is more difficult to reach consensus using OWA than using WA under the same decision weight.

(2) In Fig. 5 (a), $S(C_k) = 15$ ($k = 1, 2, 3, 4, 5$), $D(C_1) < D(C_2) < D(C_3) < D(C_4) < D(C_5)$;

and in Fig. 5 (b), $S(C_k) = 15$ ($k = 2, 3, 4, 6, 8$), $D(C_2) = D(C_6) < D(C_3) = D(C_7) < D(C_4) = D(C_8)$. Fig. 5 (a) shows that the MCC, obtained with model P_2 and P_3 , slightly fluctuate around the same fixed value as σ_w^2 increases under the same $D(C)$ and $S(C)$ values. Fig. 5 (b) shows that larger $D(C)$ values lead to lower MCC under the same $S(C)$ values. This implies that decision rules, defined by decision weights and the aggregation functions, are insensitive to the MCC, while C , defined by $D(C)$ and $S(C)$, is a main determinant in influencing the MCC.

4.3 Simulation experiment III: The impact of the consensus within the established target on MCC

This section illustrates the relation between MCC and building a consensus opinion \bar{a}^c within a desired range $[a, b]$. We firstly consider the following minimum-cost consensus model P_7 ,

$$\begin{aligned} & \min_{\bar{a}_i} \sum_{i=1}^n c_i |a_i - \bar{a}_i| \\ & \bar{a}^c = F_{OWA}(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n), \\ & \begin{cases} |\bar{a}_i - \bar{a}^c| \leq \varepsilon, i = 1, 2, \dots, n \\ a \leq \bar{a}^c \leq b \end{cases} \end{aligned} \quad (17)$$

The basic idea of Simulation experiment III is to randomly generate original individual opinions, and the values of c_i ($i = 1, 2, \dots, n$) to investigate the effect of the consensus, within the established target, on MCC through μ_c , σ_c^2 , and $[a, b]$. Simulation experiment III can be formally presented as follows.

Simulation experiment III:

Input: The number of decision makers n , the expectation of d_i 's non-cooperative behavior coefficient μ_c , the variance of d_i 's non-cooperative behavior coefficient σ_c^2 , the established threshold value ε , decision weight $w = (w_1, w_2, \dots, w_n)^T$ and the established collective solution target range $[a, b]$.

Output: The minimum consensus cost, MCC.

Step 1: Let $A = \{a_1, a_2, \dots, a_n\}$ be the original individual opinion set in model P_7 , where a_i ($i = 1, 2, \dots, n$) is randomly selected using the uniform distribution on $[0, 1]$.

Step 2: Let $C = \{c_1, c_2, \dots, c_n\}$ be the set of non-cooperative behavior coefficients in model P_7 , where c_i ($i = 1, 2, \dots, n$) is randomly generated using normal distribution $N(\mu_c, \sigma_c^2)$ subject to the constraint $c_i > 0$.

Step 3: Compute the MCC based on model P_7 .

Without loss of generality, in the simulation we consider two decision rules as follow:

$$OWA_1 = (1/n, \dots, 1/n)^T \text{ and } OWA_2 = (0, 1/(n-2), \dots, 1/(n-2), 0)^T.$$

We set different values for n , ε , μ_c , σ_c^2 , $[a, b]$ and run Simulation experiment III 10,000

times to calculate the average of the MCC values, which is shown in Figs. 6 and 7.

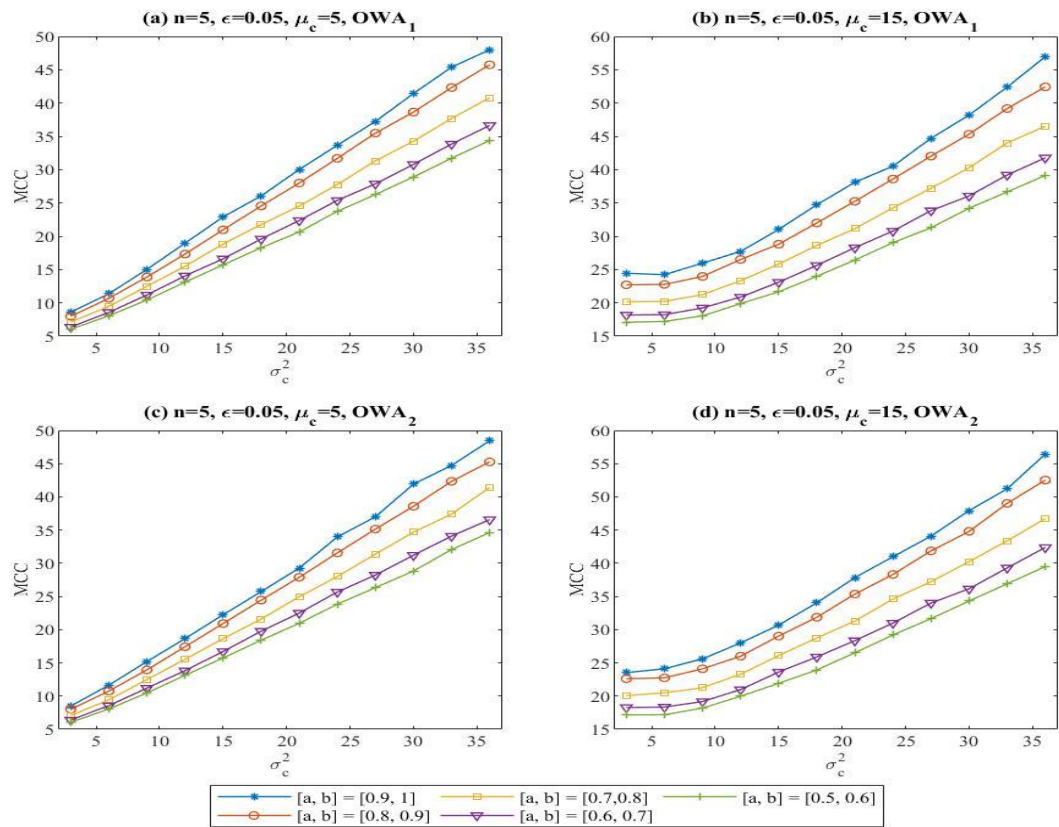


Fig. 6 Effects of $[a, b]$ and σ_c^2 on the MCC under different parameters

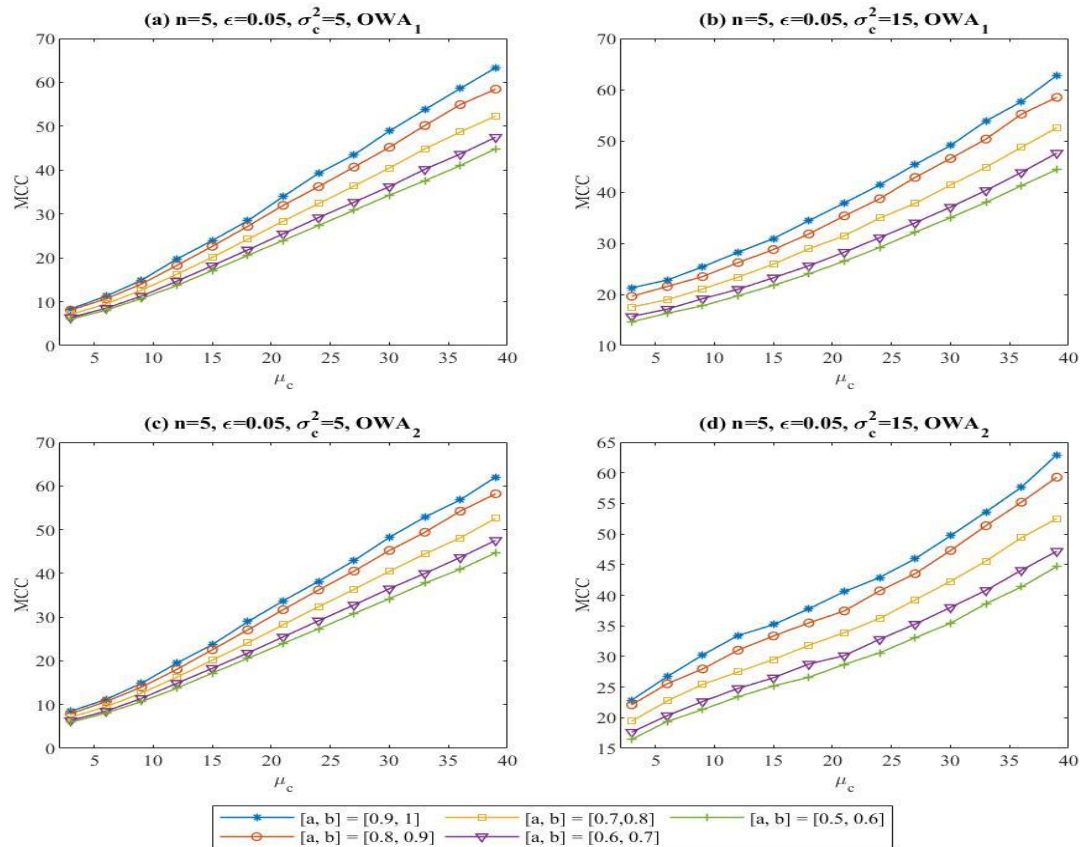


Fig. 7 Effects of $[a, b]$ and μ_c on the MCC under different parameters

From Figs. 6 and 7, we draw the following observations:

(1) Likewise to the observations previously drawn from Figs. 2 and 3, Figs. 6 and 7 also show that the MCC increases as μ_c and σ_c^2 increase under different parameters.

(2) Figs. 6 and 7 show that the MCC increases when the established collective solution target range moves up from $[0.5, 0.6]$ to $[0.9, 1]$ through $[0.6, 0.7]$, $[0.7, 0.8]$, $[0.8, 0.9]$, which means that it is more difficult to achieve an extreme collective consensus opinion. Meanwhile, this inference is more obvious under larger non-cooperative behaviors in the group, defined by μ_c and σ_c^2 .

5. Conclusion

This paper studied the impact of decision rules and non-cooperative behaviors on MCC in GDM. Theoretically, the decision rule that minimizes or maximizes MCC were derived. Furthermore, the joint impact of non-cooperative behaviors and decision rules on MCC were analyzed via simulation experiments.

In general, a preference adjustment has associated a cost and the consensus reaching resources are limited, which implies that the results obtained in this paper can provide new perspectives to understand how decision rules and non-cooperative behaviors influence the consensus cost.

Meanwhile, social network and opinion dynamics are becoming new tools to model CRP (Dong et al., 2018a; Gong et al., 2020), and we argue that it will be promising to investigate the consensus with minimum cost in social network and opinion dynamics contexts. In particular, it is very necessary to investigate the impact of decision rules and non-cooperative behaviors on MCC in social network and opinion dynamics contexts.

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